VI. *On the Tides at Malta.*

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Section I.—*Introduction.*

Admiral Sir Astley Cooper Key, K.C.B., late Superintendent of the Royal Naval College at Greenwich, and formerly Naval Commander of Malta, before leaving this country for a foreign station, placed in my hands the original record of tidal observations made by a self-registering tide-gauge at La Valetta, extending through one entire lunation and through parts of others. On the location and construction and use of the instrument I have received from Sir Cooper Key the following notes :—

"The place of the tide-gauge was about 40 or 50 yards from the entrance of the Somerset Dock (which is in the French Creek, so called by us), on the western side or left hand when entering the dock. A channel about a yard wide, and 8 or 10 yards long, led to a deep recess in which the gauge was placed. No ripple was felt from the effect either of wind or of ships moving in the neighbourhood.

"The float was a copper vessel, nearly spherical, about 8 inches in diameter ; a vertical rod attached to it passed freely through a guide, and was hinged to the end of a horizontal lever, of which the arms were so proportioned that each space marked on the tabular form between the horizontal lines [one-fourth of an inch.—G.B.A.] corresponded accurately to an inch rise or fall of the float. Care was taken to ensure that the reading on the horizontal-zero line always corresponded to a definite height of water. [I remark here that the base-line or zero of horizontal measures is printed on the sheets, and that there is no other base-line given by a pencil attached to the fixed part of the instrument frame ; I shall refer to this circumstance hereafter.—G.B.A.] I can depend on the accurate movement of the float. I have often watched the instrument.

"The cylinder on which the paper was wrapped revolved once in 24 hours [the corresponding length of paper is 20 inches.—G.B.A.], and was adjusted every morning when the paper was removed. The accuracy of the movement was however dependent on clockwork. The distance between the vertical lines on the sheets was intended to represent solar hours. The time was not taken with accuracy, but is intended to represent mean solar time at Valetta. The hours given between the lines, such as 8 a.m., &c., merely indicate that the register of the height of the water midway between the vertical lines was intended to correspond with the hours marked above the space."

The consecutive sheets were pasted together, forming a band nearly 60 feet long. For easy control of this I mounted it on two rollers. On examination the record appeared to be everywhere in general good order, with the one exception, that on April 5, 1871, there are 25 hour-spaces, as if the barrel had been slipped by hand while the pencil was in contact. I suppressed one hour-space in the part which appeared to require it, and the record of that day then accorded sufficiently with those adjoining.

Section II.—*First Treatment of the Registers for Removal of Oscillations of Short Periods, and for Measures referring to the Tides at London.*

Through many parts of the register the course of the pencil, as it would have been guided by luni-solar tides only, is disturbed by vertical oscillations (for which I anticipate the name *seiches*), with a sensibly uniform period of about 21^{m}, and with magnitude sometimes considerably exceeding that of the genuine tides. To liberate the tides from the *seiches,* the highest and lowest points of each oscillation were marked, and the intermediate point was carefully determined by compasses, and was marked; a pencil curve was then drawn through all these intermediate points, which was considered to be the true tidal curve. The number of intermediate points thus determined was nearly 1100.

The base adopted in the numerical evaluations of the height of the water (to be shortly treated) was 10 inches below the zero line of the printed register sheets; this base was adopted in order that all records of tidal elevation might have the positive sign.

For the treatment of the measures in regard to time, I determined to lay aside all consideration of harmonic sequence of arguments, and to compare every day's tide with the corresponding tide at London. The reasons for doing so were mainly these: that in fact each day's tide is sensibly independent of every other day's tide; that the tide of the Thames is a very convenient standard, because it contains no sensible trace of diurnal tide, and that in the Admiralty Tide Tables we have (as is generally understood) a fairly accurate statement of the real tides at London.

Interpreting the scale of times upon the sheets in conformity with Sir COOPER KEY's description above, and then considering those times as London times (thus introducing an error depending on difference of longitude, which is unimportant, because it can be corrected at the end), I laid down by points on the printed base-line the time of every London high water. The intervals between these points were divided each into 24 equal parts, and these were taken as corresponding to genuine tidal half-hours. At every one of these points the elevation of the tidal register was measured on the sheet. The number of these measures is about 1900.

Now, considering that the quantities of which we are in search are either constant through the tidal day (as the mean height of the water for the day) or are assumed to be periodical in the course of the tidal day (as the semidiurnal and diurnal tides in height), it is evident that we shall obtain all that is necessary for determination of the several coefficients of those quantities, by dividing the 48 measures made on each tidal day into 8 groups, each of 6 measures, and taking the mean for each group. The further treatment of these will be the subject of the next section.

The tables below exhibit the means for each of the 8 groups on each day, and these numbers are the base of all the subsequent calculations. It is to be remarked that the "Zero of Tidal Time" is 16^{m} earlier than the time of London high water, for the following reason:—The first measure of height on the tidal day was taken at the time of London high water, and the last was taken 30 tidal minutes before the following London high water. The means for the several groups therefore correspond to times 15 tidal minutes or 16 solar minutes earlier than the tidal hours (at intervals of 3^{h}), beginning with London high water. By fixing the "Zero of Tidal Time" 16 minutes earlier than London high water, the proper relation is established between the means of the groups and the subdivisions of the tidal day.

MEANS, for every group of three tidal hours, of the six half-hourly elevations of the tidal water in each of these groups, recorded at Malta, 1871 and 1872.

Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.
1871 h. m.		in.	1871 h. m.		in.	1871 h. m.		in.
Mar. 31 .. 22 22	1	10·88	April 4 .. 1 19	1	11·93	April 7 .. 3 13	1	12·23
	2	11·15		2	11·83		2	9·87
	3	9·07		3	7·15		3	5·62
	4	8·83		4	8·12		4	7·05
	5	12·62		5	12·53		5	9·18
	6	12·68		6	9·68		6	6·75
	7	9·72		7	5·85		7	4·12
	8	9·10		8	8·97		8	6·62
April 1 .. 23 42	1	12·55	April 5 .. 1 56	1	13·07	April 8 .. 3 53	1	9·70
	2	12·55		2	10·53		2	8·00
	3	8·73		3	6·32		3	5·33
	4	8·40		4	8·60		4	6·63
	5	12·62		5	12·55		5	8·57
	6	11·90		6	9·38		6	6·85
	7	7·52		7	5·93		7	5·28
	8	8·33		8	9·45		8	7·07
April 3 .. 0 34	1	12·17	April 6 .. 2 34	1	13·32	April 9 .. 4 34	1	9·68
	2	12·22		2	10·12		2	8·85
	3	8·18		3	6·37		3	6·30
	4	8·08		4	9·18		4	7·07
	5	12·52		5	12·03		5	8·55
	6	12·05		6	9·05		6	7·90
	7	7·02		7	6·02		7	6·22
	8	7·57		8	9·02		8	7·23

MEANS, for every group of three tidal hours, of the six half-hourly elevations of the tidal water in each of these groups, recorded at Malta, 1871 and 1872—*continued.*

Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.
1871 h. m.		in.	1871 h. m.		in.	1871 h. m.		in.
April 10.. 5 18	1	8·92	April 16.. 12 22	1	10·73	April 22.. 15 50	1	10·67
	2	8·52		2	7·92		2	7·70
	3	6·87		3	5·67		3	5·98
	4	6·77		4	7·40		4	9·40
	5	7·47		5	11·30		5	11·43
	6	7·55		6	9·20		6	9·35
	7	6·97		7	5·58		7	7·47
	8	7·08		8	7·87		8	10·42
April 11.. 6 8	1	8·02	April 17.. 13 7	1	11·30	April 23.. 16 22	1	11·97
	2	8·30		2	8·75		2	10·10
	3	7·80		3	5·70		3	9·03
	4	7·62		4	7·85		4	12·08
	5	8·05		5	12·17		5	13·45
	6	8·45		6	9·75		6	11·57
	7	7·83		7	6·07		7	10·22
	8	7·32		8	8·20		8	12·15
April 12.. 7 7	1	7·72	April 18.. 13 45	1	11·73	April 24.. 16 53	1	13·18
	2	8·08		2	9·35		2	11·80
	3	6·60		3	6·10		3	10·98
	4	5·38		4	10·00		4	12·53
	5	5·80		5	12·62		5	12·93
	6	6·95		6	9·15		6	11·15
	7	6·45		7	6·65		7	9·75
	8	5·52		8	10·33		8	10·62
April 13.. 8 27	1	6·98	April 19.. 14 17	1	12·80	April 25.. 17 26	1	11·77
	2	8·02		2	9·73		2	10·90
	3	6·27		3	7·13		3	10·70
	4	5·38		4	10·32		4	12·07
	5	7·57		5	13·77		5	12·95
	6	8·70		6	10·47		6	12·53
	7	6·57		7	7·00		7	11·52
	8	6·23		8	9·87		8	12·12
April 14.. 9 58	1	8·95	April 20.. 14 49	1	12·65	April 26.. 18 10	1	12·55
	2	9·05		2	9·60		2	11·62
	3	6·28		3	7·02		3	10·85
	4	6·62		4	10·65		4	11·20
	5	10·00		5	12·88		5	11·58
	6	10·03		6	9·47		6	11·42
	7	6·97		7	7·12		7	10·57
	8	7·38		8	9·77		8	10·53
April 15.. 11 23	1	10·80	April 21.. 15 18	1	11·83	April 27.. 19 2	1	11·40
	2	9·35		2	8·88		2	12·05
	3	6·28		3	7·10		3	11·85
	4	7·55		4	10·82		4	11·47
	5	11·17		5	13·27		5	12·85
	6	9·28		6	10·08		6	14·33
	7	5·80		7	7·53		7	12·77
	8	8·07		8	9·77		8	11·20

MEANS, for every group of three tidal hours, of the six half-hourly elevations of the tidal water in each of these groups, recorded at Malta, 1871 and 1872—*continued.*

Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.	Commencement of the Tidal Day, or Zero of Tidal Time, in Malta Mean Solar Time.	No. of Group.	Mean of Six Elevations.
1871 h. m.		in.	1872 h. m.		in.	1872 h. m.		in.
April 28.. 20 18	1	12·15	June 10 .. 16 50	1	13·47	July 6 .. 2 30	1	
	2	12·02		2	10·43		2	
	3	9·88		3	10·80		3	10·47
	4	8·95		4	12·85		4	14·22
	5	10·68		5	14·48		5	16·33
	6	11·35		6	12·20		6	12·33
	7	8·70		7	10·90		7	9·35
	8	8·12		8	12·73		8	13·27
April 29.. 21 42	1	11·57	June 27.. 19 20	1	13·93	July 29 .. 21 42	1	15·03
	2	11·87		2	13·35		2	13·70
	3	8·63		3	10·87		3	11·93
	4	8·85		4	11·07		4	12·47
	5	12·10		5	13·10		5	13·95
	6	12·18		6	12·22		6	11·72
	7			7	10·10		7	
	8			8	11·02		8	
1872								
June 8 .. 15 39	1		June 29.. 21 26	1	14·52	Aug. 29.. 10 16	1	16·48
	2			2	13·65		2	16·52
	3	7·85		3	10·72		3	12·83
	4	11·33		4	12·07		4	13·73
	5	12·18		5	13·77		5	16·37
	6	8·85		6	12·68		6	
	7	7·98		7	10·80		7	
	8	11·07		8	12·20		8	
June 9 .. 16 14	1	12·15	June 30 .. 22 26	1	15·70			
	2	9·67		2	15·55			
	3	9·88		3	12·57			
	4	12·87		4	12·13			
	5	13·10		5	15·35			
	6	10·82		6	14·48			
	7	9·53		7	11·02			
	8	11·75		8				

Section III.—*Treatment of the Means of the Grouped Measures, in order to express the Elevation of the Water by the Combination of Mean Height with Semidiurnal and Diurnal Tides.*

Put G1, G2 . . . G8 for the successive means of grouped measures, as exhibited in the last table, for any one tidal day. And put θ for the tidal angle, proportional to the time, whose value is 0 at the commencement of the tidal day, and is 2π at the end of it. Then the values of θ for the six measures of the first group are $\frac{\pi}{48}, \frac{3\pi}{48}, \frac{5\pi}{48}, \frac{7\pi}{48}, \frac{9\pi}{48}, \frac{11\pi}{48}$. And assume that any tidal elevation is to be expressed by

$$M+P\sin 2\theta+Q\cos 2\theta+p\sin\theta+q\cos\theta,$$

(where the second and third term express semidiurnal tide, and the fourth and fifth express diurnal tide). Then the several measures of elevation in the first group are

$$M+P\sin\frac{\pi}{24}+Q\cos\frac{\pi}{24}+p\sin\frac{\pi}{48}+q\cos\frac{\pi}{48}$$

$$M+P\sin\frac{3\pi}{24}+Q\cos\frac{3\pi}{24}+p\sin\frac{3\pi}{48}+q\cos\frac{3\pi}{48}$$

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$$M+P\sin\frac{11\pi}{24}+Q\cos\frac{11\pi}{24}+p\sin\frac{11\pi}{48}+q\cos\frac{11\pi}{48};$$

and, summing these columns vertically, and dividing by 6,

$$G1=M+\frac{P}{12\sin\frac{\pi}{24}}\left(\cos 0-\cos\frac{12\pi}{24}\right)+\frac{Q}{12\sin\frac{\pi}{24}}\left(-\sin 0+\sin\frac{12\pi}{24}\right)$$

$$+\frac{p}{12\sin\frac{\pi}{48}}\left(\cos 0-\cos\frac{12\pi}{48}\right)+\frac{q}{12\sin\frac{\pi}{48}}\left(-\sin 0+\sin\frac{12\pi}{48}\right)$$

similarly

$$G2=M+\frac{P}{12\sin\frac{\pi}{24}}\left(\cos\frac{12\pi}{24}-\cos\frac{24\pi}{24}\right)+\frac{Q}{12\sin\frac{\pi}{24}}\left(-\sin\frac{12\pi}{24}+\sin\frac{24\pi}{24}\right)$$

$$+\frac{p}{12\sin\frac{\pi}{48}}\left(\cos\frac{12\pi}{48}-\cos\frac{24\pi}{48}\right)+\frac{q}{12\sin\frac{\pi}{48}}\left(-\sin\frac{12\pi}{48}+\sin\frac{24\pi}{48}\right)$$

$$G3=M+\frac{P}{12\sin\frac{\pi}{24}}\left(\cos\frac{24\pi}{24}-\cos\frac{36\pi}{24}\right)+\frac{Q}{12\sin\frac{\pi}{24}}\left(-\sin\frac{24\pi}{24}+\sin\frac{36\pi}{24}\right)$$

$$+\frac{p}{12\sin\frac{\pi}{48}}\left(\cos\frac{24\pi}{48}-\cos\frac{36\pi}{48}\right)+\frac{q}{12\sin\frac{\pi}{48}}\left(-\sin\frac{24\pi}{48}+\sin\frac{36\pi}{48}\right)$$

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$$G8=M+\frac{P}{12\sin\frac{\pi}{24}}\left(\cos\frac{84\pi}{24}-\cos\frac{96\pi}{24}\right)+\frac{Q}{12\sin\frac{\pi}{24}}\left(-\sin\frac{84\pi}{24}+\sin\frac{96\pi}{24}\right)$$

$$+\frac{p}{12\sin\frac{\pi}{48}}\left(\cos\frac{84\pi}{48}-\cos\frac{96\pi}{48}\right)+\frac{q}{12\sin\frac{\pi}{48}}\left(-\sin\frac{84\pi}{48}+\sin\frac{96\pi}{48}\right)$$

Substituting the numerical values for the trigonometrical symbols,

$$G1=M+\frac{P}{1\cdot5663}\times(+1-0)+\frac{Q}{1\cdot5663}\times(0+1)+\frac{p}{\cdot7848}\times(+1-\sqrt{\tfrac{1}{2}})+\frac{q}{\cdot7848}\times(0+\sqrt{\tfrac{1}{2}}),$$

$$G2=M+\frac{P}{1\cdot5663}\times(0+1)+\frac{Q}{1\cdot5663}\times(-1+0)+\frac{p}{\cdot7848}\times(+\sqrt{\tfrac{1}{2}}-0)+\frac{q}{\cdot7848}\times(-\sqrt{\tfrac{1}{2}}+1),$$

$$G3=M+\frac{P}{1\cdot5663}\times(-1-0)+\frac{Q}{1\cdot5663}\times(0-1)+\frac{p}{\cdot7848}\times(0+\sqrt{\tfrac{1}{2}})+\frac{q}{\cdot7848}\times(-1+\sqrt{\tfrac{1}{2}}),$$

$$G4=M+\frac{P}{1\cdot5663}\times(0-1)+\frac{Q}{1\cdot5663}\times(+1+0)+\frac{p}{\cdot7848}\times(-\sqrt{\tfrac{1}{2}}+1)+\frac{q}{\cdot7848}\times(-\sqrt{\tfrac{1}{2}}+0),$$

$$G5=M+\frac{P}{1\cdot5663}\times(+1-0)+\frac{Q}{1\cdot5663}\times(0+1)+\frac{p}{\cdot7848}\times(-1+\sqrt{\tfrac{1}{2}})+\frac{q}{\cdot7848}\times(0-\sqrt{\tfrac{1}{2}}),$$

$$G6=M+\frac{P}{1\cdot5663}\times(0+1)+\frac{Q}{1\cdot5663}\times(-1+0)+\frac{p}{\cdot7848}\times(-\sqrt{\tfrac{1}{2}}+0)+\frac{q}{\cdot7848}\times(+\sqrt{\tfrac{1}{2}}-1),$$

$$G7=M+\frac{P}{1\cdot5663}\times(-1-0)+\frac{Q}{1\cdot5663}\times(0-1)+\frac{p}{\cdot7848}\times(0-\sqrt{\tfrac{1}{2}})+\frac{q}{\cdot7848}\times(+1-\sqrt{\tfrac{1}{2}}),$$

$$G8=M+\frac{P}{1\cdot5663}\times(0-1)+\frac{Q}{1\cdot5663}\times(+1+0)+\frac{p}{\cdot7848}\times(+\sqrt{\tfrac{1}{2}}-1)+\frac{q}{\cdot7848}\times(+\sqrt{\tfrac{1}{2}}-0).$$

As we have here eight equations from which five quantities are to be determined, it is proper to refer to the considerations of the Theory of Probable Errors. Put e for the numerical value of each of the probable errors e_1, e_2, . . . e_8 of the numbers G1, G2, . . . G8. Now to form the final equations, determining p and q, we ought in strictness to form intermediate equations by multiplying the equations above by the coefficients of p and q in those equations; and thus we should obtain results for $\frac{p}{\cdot7848}$ and $\frac{q}{\cdot7848}$, whose probable errors are $\frac{e}{\sqrt{(8-8\sqrt{\frac{1}{2}})}}$. But if, instead of this, we had multiplied each equation by $+1$ or by -1 so as to make all the coefficients for p positive (and similarly for q), we should have obtained results for $\frac{p}{\cdot7848}$ and $\frac{q}{\cdot7848}$ whose probable errors are $e\sqrt{\frac{1}{2}}$. This is greater than that found by the first process by only $\frac{1}{12}$ part. The loss of accuracy in this second process is so small, and the gain of convenience and simplicity so great, that I have not hesitated to adopt it.

We now obtain the values of M, P, Q, p, q, by the following combinations :—

$$G1+G2+G3+G4+G5+G6+G7+G8=8\,M,$$

$$G1+G2-G3-G4+G5+G6-G7-G8=\frac{8\,P}{1\cdot5663},$$

$$G1-G2-G3+G4+G5-G6-G7+G8=\frac{8\,Q}{1\cdot5663},$$

$$G1+G2+G3+G4-G5-G6-G7-G8=\frac{4p}{\cdot 7848},$$

$$G1+G2-G3-G4-G5-G6+G7+G8=\frac{4q}{\cdot 7848},$$

By these simple formulæ, which are very easy of application, the numbers in the next section are formed.

Section IV.—*Exhibition of the Tidal Elements at Malta, deduced from the Observations of each day by means of the preceding Formulæ, with Astronomical and Tidal Elements at London, for Comparison with those at Malta.*

The places of the sun and moon are taken from the Nautical Almanac; the times of London high water from the Admiralty Tide Tables. The half range of London tide is inferred from the height of high water at London, supposing the mean height of water at London to be 10 feet 1½ inch, which appears to be supported by the language of pages 151 and 155 of the Admiralty Time Tables. The Malta observations following April 29, 1871, are omitted, the series being incomplete.

POSITIONS of the Sun and the Moon, and Elements of Tides at London and Malta.

Solar and Lunar Places and London Tides.							Semidiurnal Malta Tides.								Diurnal Malta Tides.	
Time of High Water at London.	Time of Moon's preceding Transit.	Retard of High Water on Moon's preceding Transit.	Half Range of London Tide.	Moon's Declination at preceding Transit.	Sun's Declination at preceding Transit.	Mean Height at Malta.	Zero of Tidal Time.	Mean Height of Water for the Tidal Day.	Value of P.	Value of Q.	Retard of High Water on Zero of Tidal Tide.	Time of High Water.	Retard of High Water on Moon's Transit.	Half Range.	Value of p.	Value of q.
d. h. m.	h. m.	h. m.	ft. in.	° ′	° ′	inches	d. h. m.	inches	inches	inches	h. m.	h. m.	h. m.	inches	inches	inches
1871, March 31 22 38	8 17	14 21	6 4	N. 20 36	N. 4 6	10·50	31 22 22	10·50	2·08	−0·24	3 13	1 35	17 18	2·10	−0·82	−0·44
April 1 23 58	9 8	14 50	7 5	N. 17 21	4 29	10·32	1 23 42	10·32	3·27	+0·04	2 59	2 41	17 33	3·27	+0·37	−0·14
3 0 50	9 59	14 51	8 8	N. 13 0	5 15	9·98	3 0 34	9·98	3·55	0·19	2 54	3 28	17 29	3·55	0·29	−0·36
4 1 35	10 49	14 46	9 10	N. 7 47	5 38	9·51	4 1 19	9·51	3·12	1·38	2 12	3 31	16 42	3·40	0·40	+0·22
5 2 12	11 40	14 32	10 8	N. 1 58	6 1	9·48	5 1 56	9·48	2·98	2·26	1 46	3 42	16 2	3·75	0·23	0·42
6 2 50	12 31	14 19	11 3	S. 3 50	6 24	9·39	6 2 34	9·39	2·73	2·36	1 38	4 12	15 41	3·59	0·56	0·36
7 3 29	13 24	14 5	11 8	S. 9 45	6 46	7·68	7 3 13	7·68	2·87	1·71	1 59	5 12	15 48	3·33	1·59	0·83
8 4 9	14 20	13 49	11 7	S. 15 5	N. 7 9	7·18	8 3 53	7·18	1·72	1·28	1 47	5 40	15 20	2·13	0·37	0·53
9 4 50	15 17	13 33	11 2	S. 19 22	7 31	7·73	9 4 34	7·73	1·60	0·64	2 17	6 51	15 34	1·73	0·39	0·42
10 5 34	16 17	13 17	10 4	S. 22 16	7 53	7·52	10 5 18	7·52	0·94	+0·07	2 52	8 10	15 53	0·94	0·39	+0·56
11 6 24	17 17	13 7	8 11	S. 23 34	8 15	7·93	11 6 8	7·93	0·44	−0·27	4 3	10 11	16 54	0·51	0·02	−0·09
12 7 23	18 16	13 7	7 9	S. 23 13	8 37	6·57	12 7 7	6·57	0·90	−0·72	4 17	11 24	17 8	1·15	+0·60	+0·60
13 8 43	19 13	13 30	6 10	S. 21 22	8 59	6·97	13 8 27	6·97	1·33	−0·67	3 53	12 20	17 7	1·48	−0·48	−0·03
14 10 14	20 7	14 7	7 1	S. 18 17	9 21	8·17	14 9 58	8·17	2·11	+0·12	2 54	12 52	16 45	2·11	−0·69	−0·11
15 11 39	20 57	14 42	7 10	S. 14 14	N. 9 42	8·54	15 11 23	8·54	2·53	1·35	2 4	13 27	16 30	2·87	−0·07	−0·05
16 12 38	21 43	14 55	8 9	S. 9 32	10 4	8·21	16 12 22	8·21	2·48	1·75	1 50	14 12	16 29	3·05	−0·44	−0·29
17 13 23	22 28	14 55	9 7	S. 4 41	10 25	8·73	17 13 7	8·73	2·78	1·82	1 54	15 1	16 33	3·30	−0·51	−0·22
18 14 1	23 11	14 50	10 1	N. 0 30	10 46	9·50	18 13 45	9·50	1·92	2·64	1 12	14 57	15 46	3·25	−0·31	+0·04
19 14 33	23 53	14 40	10 3	N. 5 35	11 7	10·14	19 14 17	10·14	2·44	2·44	1 30	15 47	15 54	3·45	−0·22	−0·45
20 15 5	0 36	14 29	10 4	N. 10 23	11 28	9·90	20 14 49	9·90	1·97	2·50	1 17	16 6	15 30	3·19	+0·13	−0·17
21 15 34	1 19	14 15	10 2	N. 14 33	11 48	9·91	21 15 18	9·91	1·73	2·37	1 13	16 31	15 12	2·94	−0·40	−0·64
22 16 6	2 4	14 2	9 11	N. 18 14	N. 12 8	9·06	22 15 50	9·06	1·15	2·24	0 55	16 45	14 41	2·52	−0·96	+0·02
23 16 38	2 50	13 48	9 7	N. 21 5	12 29	11·33	22 16 22	11·33	0·71	1·71	0 45	17 7	14 17	1·85	−0·82	−0·33
24 17 9	3 38	13 31	9 0	N. 22 55	12 48	11·62	26 16 53	11·62	1·02	1·09	1 26	18 19	14 41	1·49	+0·79	−0·44
25 17 42	4 28	13 14	8 1	N. 23 45	13 8	11·82	25 17 26	11·82	0·34	0·64	0 56	18 22	13 54	0·73	−0·72	−0·38
26 18 26	5 18	13 8	7 2	N. 23 24	13 28	11·30	26 18 10	11·30	0·79	+0·27	2 22	20 32	15 14	0·83	+0·42	+0·04
27 19 18	6 8	13 10	6 6	N. 21 51	13 47	12·24	27 19 2	12·24	0·66	−0·80	4 41	23 43	17 35	1·04	−0·86	−0·60
28 20 34	6 58	13 36	6 1	N. 19 7	14 6	10·23	28 20 18	10·23	2·07	−0·40	3 22	23 40	16 42	2·10	+0·82	+0·02

The purport of the deductions from these computations will be more distinctly seen on taking the means for each eighth part of the lunation. This operation, which required for the means the subdivision of each 7 days period into two equal parts of $3\frac{1}{2}$ days each, was thus performed:—The mean of days 1, 2, 3 was taken, and the mean of days 1, 2, 3, 4 was taken, and the mean of these two means was adopted as the fitting mean for the first subdivision of $3\frac{1}{2}$ days. Then the means of days 4, 5, 6, 7 and of days 5, 6, 7 were taken, and their mean was adopted as mean for the second subdivision of $3\frac{1}{2}$ days.

MEANS, for Subdivisions each one-eighth part of Lunation, of the Principal Elements for Positions of Sun and Moon, London Tides, and Malta Tides.

Subdivision.	London Tides.				Position of Sun and Moon.		Malta Semidiurnal Tides.						Malta Diurnal Tides.	
	London Retard of High Water on Moon's Transit.	London Retard diminished by $14^h\ 3^m$.	London Tide Half Range.	London Tide Half Range diminished by 9 ft. 1 in.	Moon's Declination.	Sun's Declination.	Malta Mean Height.	Malta Mean Height diminished by 9·33 in.	Malta Retard of High Water on Moon's Transit.	Malta Retard diminished by $16^h\ 4^m$.	Malta Tide, Half Range.	Malta Tide Half Range diminished by 2·39 in.	Value of *p*.	Value of *q*.
	h. m.	h. m.	ft. in.	ft. in.	° ′	° ′	inches.	inches.	h. m.	h. m.	inches.	inches.	inches.	inches.
I.	14 41	+0 38	7 10	−1 3	N. 15 50	N. 4 45	10·18	+0·85	17 20	+1 16	3·03	+0·64	+0·01	−0·25
II.	14 22	+0 19	11 0	+1 11	S. 2 25	6 17	8·94	−0·39	15 56	−0 8	3·54	+1·15	+0·92	+0·50
III.	13 30	−0 33	10 9	+1 8	S. 19 30	7 37	7·54	−1·79	15 47	−0 17	1·72	−0·67	+0·34	+0·43
IV.	13 31	−0 32	7 6	−1 7	S. 21 17	8 53	7·33	−2·00	16 59	+0 55	1·45	−0·94	−0·17	+0·12
V.	14 50	+0 47	8 11	−0 2	S. 5 52	10 10	8·62	−0·71	16 24	+0 20	3·10	+0·71	−0·34	−0·16
VI.	14 31	+0 28	10 3	+1 2	N. 8 58	11 22	9·92	+0·59	15 34	−0 30	3·20	+0·81	−0·18	−0·37
VII.	13 43	−0 20	9 4	+0 3	N. 21 8	12 34	10·82	+1·49	14 27	−1 37	1·80	−0·59	−0·38	−0·27
VIII.	13 18	−0 45	6 10	−2 3	N. 21 45	13 41	11·33	+2·00	16 8	+0 4	1·25	−1·14	+0·02	−0·21
No. of Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Section V.—*Examination of the Tidal Results for Malta obtained in the last Section.*

The first result claiming attention is that for the daily mean heights of the water abstracted in columns 7 and 8, column 8 giving the difference of the value for each eighth part of lunation from the mean of all. And the numbers in column 8 are to be compared with those in column 5 for the moon's declination. There cannot be a doubt that they exhibit an accurate luni-menstrual variation of height, depending on the moon's declination. Viewing the extreme slowness of the formation of this inequality, I think it probable that the amount at Malta is very little smaller than that on the Atlantic coasts of Spain and Morocco, and that its epoch is very little in retard, and that thus we have gained a term in the expression for the ocean-tides which it might not have been easy to obtain from direct observation.

The second result is that for the retard of high water on moon's transit in column 9; the difference of each number from the mean of all, in column 10, shows the semi-menstrual inequality of time at Malta, which is perfectly well defined. On comparing it with the tabular semi-menstrual inequality of time at London, in column 2, it appears that its amount is nearly the same as that at London (at first sight it appears larger, but that is perhaps the effect of small irregularities), and its epoch is certainly about three days earlier than the tabular epoch for London. How far the London retardation may depend on friction (which was shown in the treatise on Tides and Waves in the 'Encyclopædia Metropolitana' to be the probable cause of the "age of tide") I am unable to say.

The mean retard of semi-menstrual tide on the moon's transit appears to be about $16^h\ 4^m$, which may be taken as the "Establishment" of Malta.

The third result is that for the semi-range in height of Malta tide in column 11, and its variations in column 12. And here again is a perfectly clear semi-menstrual inequality. This is to be compared with the tabular London semi-menstrual inequality in column 4. In regard to the proportion of inequality of height to the whole semi-range in height, if we divide the sum of inequalities without regard of sign by the mean semi-range, the quotient for Malta is 2·78, while that for London is 1·13 (supposing that no error has been committed in regard to the zero of London heights). I cannot explain this difference. In regard to the epoch of inequality, the heights agree with the times in making the epoch for Malta about three days earlier than that for London.

It appears curious that such results should be obtained, apparently with certainty, in a single lunation from these miniature tides, in which the semi-range, even for spring tides, is less than a hand-breadth.

The remaining results relate to the diurnal tides, and these are not so clear as those which we have already treated. We may first consider what we ought to expect in diurnal tides. Referring everything to lunar time (not quite the same thing as tidal time) we should expect that the lunar diurnal tide would occur at the same lunar hour every day, and with a coefficient periodical in a month. The solar diurnal tide, if sufficiently important to be considered, would have a coefficient varying slowly, and the epoch of its daily tide would advance on the moon's, gaining 2π in a month (approximately). So that putting M and S for certain coefficients depending on the moon and sun, the first nearly constant and the latter not varying much in the lunation, and R and r for certain angles, both periodical in a lunation (nearly), we should have for approximate height at any time—

$$\begin{aligned}&\text{M} \sin \text{R} \cos (\text{moon's hour angle}-\text{A})\\&+\text{S} \cos (\text{sun's hour angle}-\text{A}),\end{aligned}$$

$$\begin{aligned}\text{or } &\text{M} \sin \text{R} \cos (\text{moon's hour angle}-\text{A})\\&+\text{S} \cos (\text{moon's hour angle}+r-\text{A}),\end{aligned}$$

$$\begin{aligned}\text{or } &(\text{M} \sin \text{R}+\text{S} \cos r) \cos (\text{moon's hour angle}-\text{A})\\&-\text{S} \sin r \sin (\text{moon's hour angle}-\text{A}).\end{aligned}$$

Each of the factors, it is to be remarked, is periodical in a lunation.

It will be remembered that p is the factor of the sine of tidal angle measured from the tidal zero of each day, and that q is the factor of the cosine of the same angle. Now if we connect the expression $p \sin \theta+q \cos \theta$ into the form B. cos $(\theta-\text{A})$, where θ is the tidal angle from the zero of tidal time, we obtain

	inch.	h. m.
I.	$+0{\cdot}25 \cos (\theta-11\ 51)$,	
II.	$+0{\cdot}90 \cos (\theta-\ 3\ 45)$,	
III.	$+0{\cdot}55 \cos (\theta-\ 2\ 33)$,	
IV.	$+0{\cdot}21 \cos (\theta-20\ 21)$,	
V.	$+0{\cdot}38 \cos (\theta-16\ 19)$,	
VI.	$+0{\cdot}41 \cos (\theta-13\ 44)$,	
VII.	$+0{\cdot}47 \cos (\theta-15\ 38)$,	
VIII.	$+0{\cdot}21 \cos (\theta-11\ 38)$.	

And if instead of θ we introduce ϕ, the tidal angle from the moon's transit, where $\theta=$

$$\phi-\text{London retard of high water on moon's transit}+16^{\text{m}},$$

we obtain for the diurnal tide

	inch. h. m.	inch. h. m.
I.	$+0{\cdot}25 \cos (\phi-\ 2\ 16)$	
II.	$+0{\cdot}90 \cos (\phi-17\ 51)$	$=-0{\cdot}90 \cos (\phi-5\ 51)$
III.	$+0{\cdot}54 \cos (\phi-15\ 47)$	$=-0{\cdot}54 \cos (\phi-3\ 47)$
IV.	$+0{\cdot}21 \cos (\phi-\ 9\ 36)$	$=-0{\cdot}21 \cos (\phi-2\ 24)$
V.	$+0{\cdot}38 \cos (\phi-\ 6\ 53)$	
VI.	$+0{\cdot}41 \cos (\phi-\ 3\ 59)$	
VII.	$+0{\cdot}46 \cos (\phi-\ 5\ \ 5)$	
VIII.	$+0{\cdot}21 \cos (\phi-\ 0\ 40)$	

Remarking the smallness of the tide (which is little more than a finger-nail's breadth) it may be considered that the tides II., III., IV. present that opposition of sign to VI., VII., VIII. which they ought to maintain, and that the term depending on S may explain some part of the remaining discordance.

There is one mechanical consideration applying to the register of the diurnal tide which does not apply in the same degree to the semidiurnal tide. A single sheet at a time, as I understand, was placed on the revolving barrel; and it is not easy to attach this without the risk of a little inclination, which would produce the appearance of a diurnal tide; and there was no base-line produced by a pencil on the fixed part of the instrument which would give evidence on this point. No error, however, appears to have been produced in the mean height for the day.

Section VI.—*On the "Seiches" or Non-Tidal Undulations of Short Period at Malta.*

I have mentioned above the fluctuations of short period observable on the tidal record made by the self-registering tide gauge, and the methods of eliminating their effect in the treatment of the tides. During the month of the tidal discussions these fluctuations are small, but in some following seasons of 1871 and 1872 they became more important.

Their general characteristic is; that they are simple harmonic curves (excepting in the larger undulations in 1872, when their heads are sometimes notched, as by the intermixture of small waves originating from different causes); that they recur with great but not perfect uniformity at intervals of about 21 minutes of time; and that they continue for many hours at a time, sometimes for entire days; that their magnitude is very variable, sometimes small, sometimes amounting to ± 6 inches (or producing a range of 12 inches), much exceeding that of the luni-solar tides.

These fluctuations, occurring in a sea usually so tranquil as regards movements of slow character, were naturally known at Malta, and were not unfrequently attributed (conjecturally) to volcanic disturbances of Stromboli. I knew, however, that similar fluctuations were recorded by a self-registering tide-gauge at Swansea (see the 'Encyclopædia Metropolitana,' "Tides and Waves"), and was persuaded that they were due to some hydrodynamic cause.

At this time I received from Dr. FOREL, of Lausanne, an account of his remarkable investigations on the undulations locally known as "Seiches," in the Lake of Geneva, and on similar but less conspicuous fluctuations in other lakes of Switzerland. ('Annales de Chimie et de Physique,' 5^{me} serie, tom. ix., 1876; 'Archives des Sciences de la Bibliothèque Universelle,' August and December, 1876; May, 1877.) On comparing the fluctuations on the tidal sheets with the descriptions by Dr. FOREL, and above all with his engraved diagrams, it was impossible to doubt that they are phenomena of the same class. Every form delineated by Dr. FOREL is to be found, I believe, upon the Malta sheets; but I think that the simple harmonic curve of longest period occurs more frequently at Malta than in Switzerland. The origin assigned by Dr. FOREL is, I think, most certain; that they are waves originally caused by winds; but that they are reflected from one side and another of the limited sea, and thus become stationary waves. The waves forming the seiches of Malta are reflected, I suppose, from the shores of Sicily and Africa.

On examining the chart of those seas, it appears that a large bank, covered by comparatively shallow water, projects from the African coast, bounded roughly by a line nearly from Cape Bon to the island Linosa; and that between that bank and a narrower bank on the Sicilian side there is a broad sea-channel of approximately uniform width and of extremely deep water. I imagine that the reflection of undulations takes place principally, not at the shores of the land, but at the edges of the banks bounding the deep water. I have not yet ventured on a numerical calculation, but in rough estimate it appears to me that the breadth and depth of this sea would hydrodynamically explain the return of waves at periods of 21^{m}. Such waves, once created, would be propagated to regions of the sea somewhat beyond those in which they are formed.

For closing this account, I think that nothing more is required than to append a table descriptive of the seiches on the days in which they are large enough to attract attention. I have thought it sufficient to state the number of undulations occurring in a group between two limits of time, thus giving the average length in time of an undulation, and to state the greatest elevation or depression measured from the luni-solar tidal curve which occurs in the group. The limits of the groups are quite arbitrary, having been determined in most instances for convenience by the length of the sheets of paper on which the measures were transcribed.

From—	To—	Interval in Time.	Number of Waves.	Mean Duration of One Wave.	Largest Rise or Fall in the Group.
d. h. m.	d. h. m.	h. m.		m.	inches.
1872. June 9 6 10	1872. June 9 13 15	6 55	18	23	2·2
13 15	20 45	7 30	20	22·5	1·7
20 45	10 4 10	7 25	19	23·4	6·4
10 4 10	12 10	8 0	20	24	5·0
12 10	20 15	8 5	19	25·5	6·8
20 15	11 4 10	7 55	20	23·75	7·8
11 5 20	10 15	4 55	14	21·1	8·2
10 15	17 10	6 55	19	21·8	4·2
17 10	21 10	4 0	12	20	2·1
21 20	12 1 20	4 0	12	20	2·8
28 0 5	28 6 55	6 50	18	22·8	1·5
6 55	14 35	7 40	20	23	0·9
14 35	20 15	5 40	19	17·9	0·9
20 15	29 0 45	4 30	12	22·5	1·5
29 22 10	23 45	1 35	4	23·8	1·2
30 0 5	30 4 10	4 5	10	24·5	1·8
4 10	11 45	7 35	19	23·9	3·0
11 45	19 0	7 15	20	21·75	3·2
19 0	July 1 3 15	8 15	19	26·1	1·8
July 1 3 15	11 10	7 55	19	25	1·6
11 10	18 40	7 30	19	23·7	1·7
18 40	20 55	2 15	6	22·5	1·2
6 5 10	6 9 30	4 20	13	20	1·2
9 30	17 0	7 30	19	23·7	1·9
17 0	21 25	4 25	12	22·1	2·2
21 40	23 50	2 10	6	21·7	2·4
7 0 10	7 6 30	6 20	16	23·75	4·7
30 0 10	30 8 35	8 25	18	28·1	1·0
8 35	18 50	10 15	19	32·4	1·3
19 10	20 30	1 20	3	26·7	0·8
August 28 21 30	August 29 5 45	8 15	20	24·75	2·4
29 5 45	13 15	7 30	19	23·7	4·5
13 15	15 10	1 55	5	23	4·5

APPENDIX.

My first knowledge of the Seiches at Swansea was derived from a letter of J. W. G. GUTCH, Esq., dated "Swansea, Feb. 18, 1838," of which the following is an extract:—

"I have some very singular curves marked [by the self-registering tide-gauge] in last month, and should much like to hear your explanation of them. They appear thus [with a rough diagram], and generally the irregularities occur at the top of the tide, and apparently at intervals of about a quarter of an hour or twenty minutes; they seem to be large tidal waves."

In my reply, dated February 27, 1838, I said: "The small irregularities of height which you mention puzzle me extremely. I do not see how they can come from the tides; whether from any strange reflections of waves in the bay or across the channel I cannot tell."—G. B. AIRY, *October* 4, 1877.